TEMPORAL TRENDS OF WATER CONTENT UNDER GRASSLAND: CHARACTERIZATION USING THE MULTIFRACTAL APPROACH

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RESUMEN. Se empleó la técnica de la reflectometría en el dominio de frecuencias (FDR) para monitorear en continuo el contenido hídrico del suelo bajo pradera en condiciones de clima atlántico. En este trabajo se analiza una serie de datos medida a seis profundidades (10, 20, 40, 60, 90 y 120 cm) a escala horaria durante el año 2009. Los datos obtenidos con los sensores de capacitancia permitieron efectuar una descripción detallada del régimen hídrico del suelo y del uso del agua por la pradera durante las estaciones húmeda y seca y también fueron útiles para determinar la profundidad que alcanza la actividad del sistema radicular. Se comprobó que el análisis multifractal es un método capaz de diferenciar entre series de contenido hídrico medidas a sucesivas profundidades. El índice de multifractalidad, medido mediante la amplitud de la dimensión generalizada, D_q, disminuye conforme aumenta la profundidad del suelo.

ABSTRACT. The frequency domain reflectometry (FDR) technique was used to continuously monitor soil water content under grassland in Atlantic climatic conditions. In this work, we analyzed a data set measured at six depths (10, 20, 40, 60, 90 and 120 cm) on an hourly basis during 2009. Data gathered with the capacitance sensors provided a detailed description of the soil water regime and soil water use by grassland crop during the dry and the wet seasons of the year and also were useful to determine the depth of the root system activity. The multifractal approach was sensitive for differentiating soil water content series measured at successive depth layers. Multifractality index, measured by the width of the generalized dimension, D_q, decreased with increasing soil depth.

1.- Introduction

Soil water sensors may be used in applications such as crop production research, water budget assessment, precision agriculture, environmental monitoring, and irrigation scheduling. Throughout the past decade, advances in sensor design, coupled with computer technology, resulted in an increase in the number and types of sensors available for water content measurement. Various recent developed sensors are designed for continuous monitoring. Many of the new sensors measure variations in the soil dielectric constant, as water content changes in the soil. The main techniques used today for water content measurement such as Frequency Domain Reflectrometry (FDR) or Time Domain Reflectrometry (TDR) are capacitance methods based on the measure of the soil dielectric constant. FDR sensors transmit a radio frequency through the soil using two antennae in the form of copper bands, and the water content is determined by the change in frequency of the signal transmitted between the antennae. The degree of frequency shift in FDR is related to soil water content.

Fractal models have been widely applied in soil science since the 1990's, as soil properties may be interpreted through concepts of scale invariance, fractals and multifractals. For example, the water retention characteristics have been described using several different fractal models (Tyler and Wheatcraft, 1990; Perfect et al., 1996). More recently, interest has turned to multifractal analysis of soil properties (Caniego et al., 1995; Vidal Vázquez et al., 2008). Many complex processes occurring in soils are often characterized by inhomogeneity, a feature that can be described by the scaling properties.

A monofractal process is homogeneous in the sense that its scaling properties can be characterized by a single singularity exponent. In contrast, multifractals require many indices to characterize their scaling properties and they can be decomposed into many – possibly infinitely – subsets characterized by different scaling exponents. It follows that multifractals are intrinsically more complex and inhomogeneous than monofractals.

Several authors carried out multifractal studies of inhomogeneous temporal data series. For example Jiménez-Hornero et al., 2010 described ozone time series using the multifractal formalism. The use of multifractal tools to understand soil physical variables means that these can be viewed as a singular statistical distribution that is reasonable to explain as a multifractal measure. Soil water content behaves like an inhomogeneous geophysical signal characterized by a spiky dynamics, with sudden and intense bursts of high frequency activity, mainly at the soil surface layers. Until now, however, continuously monitored soil water content has not been analyzed using the multifractal approach. Therefore, the aim of this study was to describe soil water dynamics under grassland in Atlantic climate and to assess multifractality of soil water content time series.

2. - Material and Methods

2.1.- Study area and experimental setup

The study was conducted at the experimental farm of the Centre for Agriculture Investigation of Mabegondo (CIAM) located in A Coruña, Spain. Soils are Umbrisols (WRB, 1994) developed over basic schist of the Ordenes complex, characterized by its loamy to silty-loam texture.

The volumetric soil water content was continuously monitored through the soil profile in a grassland field using a capacitance probe (EnviroSCAN from Sentek, Australia), based on frequency domain reflectometry (FDR) technique (Arauzo et al., 2005; Sentek, 2000). Six sensors were installed on a access tube at the 0.10, 0.20, 0.40, 0.60, 0.90 and 1.20 m depth and connected to a data logger. The probes were properly maintained for recording soil water content at hourly intervals during all the year 2009.

Climate is temperate Atlantic with moderate summer water deficit. Agro-meteorological data were recorded by an automated station located within the experimental station. During 2009, mean temperature was 13.1 ºC, whereas yearly rainfall was 1373.1 mm.

2.2.- Multifractal analysis

Multifractal analysis of time series of soil water content supported on a given time interval I = [a,b] requires a set of different non-overlapping boxes or subintervals of I with equal length. A common choice is to consider dyadic scaling down (e.g. Caniego et al., 2005; Vidal Vázquez et al., 2008), which means successive partitions of the support in k stages (k=1,2,3…). This generates a number of cells $N(\delta) = 2^k$ of characteristic time resolution, $\delta = L \times 2^{-k}$, covering the initial interval I.

Application of multifractal formalism to time series has been described recently (Jiménez-Hornero et al., 2010), so we summarize the computational techniques used in this study. The time interval of FDR series, L, was from 1 hour to and year and the minimum time resolution, δ_{ini}, was selected so that every initial interval contained at least one hourly averaged soil moisture data, θ_{ini}. Following this approach, the probability mass distribution, $p_i(\delta)$, at time resolution δ was estimated as a proportion according with:

$$p_i(\delta) = \frac{\theta_i(\delta)}{\sum_{j}^{n_{ini}} (\theta_{ini})_j} \tag{1}$$

where θ_i is the water content of the i^{th} box or interval and n_{ini} is the number of initial intervals with mean moisture content θ_{ini}.

To analyze the multifractal spectrum of the probability mass function, $p_i(\delta)$ the moment method was used (Evertsz and Mandelbrot, 1992, Chhabra et al., 1989). First, the partition function $\chi(q,\delta)$ was estimated from the $p_i(\delta)$ values:

$$\chi(q,\delta) = \sum_{i=1}^{n} p_i(\delta)^q \tag{2}$$

The partition function scales with the box size, δ, as:

$$\chi(q,\delta) \propto \delta^{-\tau(q)} \tag{3}$$

where τ(q) is the mass exponent or scaling function of order q, which can be obtained by plotting $\chi(q,\delta)$ versus δ on log-log diagrams as the limit when δ w.

Multifractal sets can also be characterized by their spectrum of generalized dimension, D_q, which can be directly obtained from the mass exponent, τ_q, and th order q as:

$$D_q = \tau(q)/(1-q)\,,\ (q\neq 1) \tag{4}$$

For the particular case where q=1 equation (4) becomes indeterminate; so l'Hôpital's rule is applied that leads to the following equation:

$$D_1 = \lim_{\delta \to 0} \frac{\sum_{i=1}^{n} p_i(\varepsilon)\log[p_i(\delta)]}{\log(\delta)} \tag{5}$$

The generalized dimensions, D_q for $q = 0$, $q = 1$ and $q = 2$, are known as the capacity, the information (Shannon entropy) and correlation dimensions, respectively. In homogeneous structures D_q are close to one another, whereas for a monofractal structure they are equal.

In many works, the main multifractality properties have also been described by several parameters derived from D_q and τ_q (Cheng et al., 1999; Vidal Vázquez et al., 2008). A difference $w = (D_{-10}-D_{10})$ has been defined as the width of the generalized dimension spectra and suggested to be an important multifractal parameter. In the same way, other widths or differences for several q moments, as for example (D_0-D_2) also characterize the multifractality of the soil water content time series.

3.- Results and Discussion

3.1.- Patterns of soil water content under grassland

Fig. 1 shows daily rainfall together with soil water content time series at six successive depths with the multisensor capacitance probes in 2009. Contrasting rainfall patterns and soil water regimes during the moist and the dry periods can be clearly observed. So, by the end of January and beginning February the soil profile is quasi saturated from 10 to 120 cm depth and groundwater level was observed at the bottom of the soil profile within this layer. Thereafter, a relatively rapid decrease in soil water content was observed at all depths, which means that the phreatic level falls below the maximum depth recorded by redistribution of gravitational water. Noteworthy, the highest water content recorded in the field (38-39 $cm^3.100cm^3$) is not very different from the water content measured in the laboratory near saturation by the Richard apparatus (Mestas Valero et al., 2011).

During the dry periods a rapid decrease of soil water content was detected both at 10 and 20 cm depth, which was more evident at the first soil layer. Sensors below 20 cm depth; showed a lesser trend to decrease soil water content with increasing time without rainfall. In summer months clearly various periods of soil water deficit were detected;, followed by recharge after rainfall, whose importance was higher at the 10 and 20 cm profile levels. Notice also that water withdrawal under grassland occurs mainly at the top 0-40 cm layer of the soil, so that it is much lesser at the 40 cm depth, and becomes negligible at 90 or 120 cm depth.

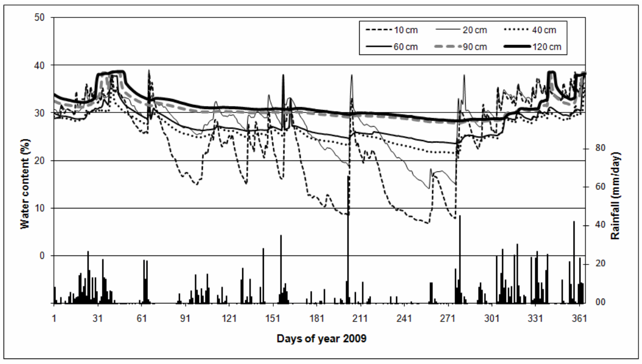

Fig. 1. Daily rainfall and soil water content continuously recorded at 10, 20, 40, 60, 90 and 120 cm depth.

Finally, in October and November generalized recharge of all the depths of the non saturated profile occurs, so that in December again a quasi stationary saturated state is reached.

3.2.- Multifractality of soil water content time series

First, the log- log plots of the normalized measures $\chi(q,\delta)$ versus measurement scale,δ, were studied to find out wether the temporal pattern of soil moisture obeys power law scaling.

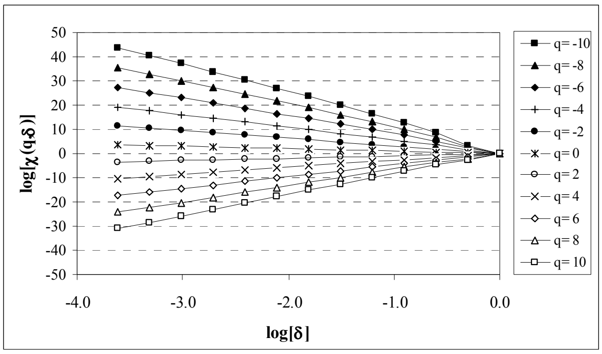

Fig. 2. Log-log plots of the partition function, $\chi(q,\delta)$ versus the time resolution, δ, at the 10 cm depth.

Fig. 2 shows, as an example, the the log-log plots of the partition function of θ versus the time resolution in the range from d = 2 data (2 hours) to d = 213 data (8192 hours) for the 10 cm depth. For all the studied statistical moments (q=-10 to q =10) the logrithm of the normalized measures varied lineraly with the logarithm of the measurements scale ($r^2 > 0.98$). The distribution of a measure is considered as a fractal (mono- or multifractal) when the moments obey power laws, which is interpreted as self-similar behaviour.

The scaling properties observed in Fig. 2 for the 10 cm depth and those for successive depths (data no shown) can be further characterized by determining if it is simple (monofractal) or multiple (multifractal) scaling types. Mathematically, the multifractal property can be completely determined only by the entire spectrum of the three three multifractal functions, i. e. mass exponent, τ_q, generalized dimension, D_q, and singularity spectrum, $f(\alpha)$, (not used in this work). Some characteristic values of these functions, however, portray the main characteristics of multifractal spectrum and they will be shown to be useful for describing multifractality. In this study selected indicator parameters were calculated from D_q. Note that for a monofractal distribution values of D_1 and D_2 become similar to D_0, whereas, if $D_0 > D_1 > D_2$ the distribution has a tendency to multifractal type of scaling.

Table 1. Values of information dimension (D_1), correlation dimension (D_2) and the width of the D_q spectra, w, ($w = D_{-10}-D_{10}$) as a function of soil depth.

Depth	**D_1**	**D_2**	**w**
10 cm	0.9908 ± 0,0016	0.9845 ± 0,0028	0.1324
20 cm	0.9972 ± 0.0004	0.9947 ± 0,0007	0.0604
40 cm	0.9987 ± 0.0002	0.9972 ± 0,0005	0.0259
60 cm	0.9987 ± 0.0002	0.9973 ± 0,0005	0.0262
90 cm	0.9989 ± 0.0002	0.9977 ± 0,0005	0.0221
120 cm	0.9989 ± 0.0002	0.9976 ± 0,0004	0.0000

The capacity or box-counting dimension, D_0, was not significantly different from 1.00 (data not shown), which correspond to an Euclidean support for a one-dimensional temporal series and means that all the partitions taken into account "contained object". Values for D_1 and D_2, are listed in Table 1, together with those of the width of the generalized dimension, w, for moments q=-10 and q = 10, which can be considered as a mutifractality index.

Fig. 3 shows generalized dimensions computed in the moment range from q= -10 to q =10 for the soil moisture contents at the six depth measured. At the 10 and 20 cm depth typical sigma shaped curves with an inflexion point at the point q =0 vs. $D_q = 1$ and less curvature for $0 < q < 10$ are obtained. At the 40 cm depth and below D_q functions are quasi linear, so that the departure from 1 both for $q >> 0$ and $q << 0$ is almost negligible. A sigma-shaped D_q is taken as an indication of the multifractality of the measure, whereas quasi-linear spectra are close to monofractals.

Thus, the entropy or information dimension, D_1, was closer to 1.00 (i.e. D_1 approached D_0) as the sensor depth increased. The value of D_1 is a good index of the degree of heterogeneity in temporal distribution of a measure. The closer the D_1 value to D_0, the more homogeneous is the distribution of the measure.

Also width indices obtained from the generalized dimension function, such as (D_0 - D_2) or (D_{-10} - D_{10}) decreased with soil depth. The higher width of the generalized dimension spectra near the soil surface are in accordance with the spiky nature of the water content time series, implying a multiple scaling nature of this variable at shallow depths.

Results obtained form the generalized dimension analysis agree with those from the mass exponent function calculated with Eq. (3), which also indicate a lesser multifractality of soil water content time series as the soil depth increases (data not shown). Therefore, the high temporal heterogeneity of the soil water content of the soil surface layers is associated with a higher degree of multifractality, whereas the more homogeneous annual trends of soil water content of the deeper layers result in a lower degree of multifractality or even in monofractal behaviour.

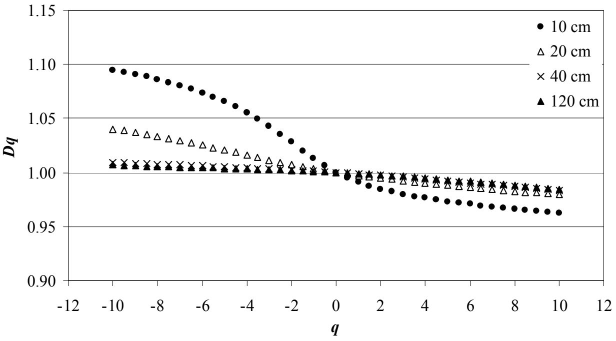

Fig. 3. Generalized dimension spectra of soil moisture content at 10, 20, 40 and 120 cm depth

4.- Conclusions

Continuous FDR records of soil water content at various depths allowed a reliable description of soil water regime under grassland in temperate Atlantic climatic conditions.

The logrithm of the normalized measures varied lineraly with the logarithm of the measurement scale for all the studied depths in the moment range -10 < q < 10, meaning that the measure, i.e. the time series of soil water content obey power laws and should be considered as a fractal.

The scaling properties of soil water content time series measured in the grassland root zone could be fitted reasonably well with multifractal models, whereas below the root zone a trend to monofractal behaviour of these series was observed. Thus, multifractal analysis allowed discriminations between different patterns of yearly water content series as a function of soil depth.

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